APPROXIMATE SOLUTION BY FRONTAL METHOD OF TWO-DIMENSIONAL PROBLEM ON FORMATION OF CRATER IN A METAL BY A LASER PULSE

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The two-dimensional formation of a crater in a metal by a laser pulse is studied by the frontal method. The problem is reduced to a set of transcendental equations for the evaporation constants and to an equation of the first order for the frontal surface. Calculations are made for Al, Cu, and Fe targets for laser energy flux densities from 10^6 to 10^8 W/cm^2.

A light flux of radius b with a prescribed radial energy density distribution q(r) (a monotonically decreasing function) falls upon the surface of a metal. The absorption of energy on the surface of the metal results in the melting and evaporation of the latter. It is required to find the rate of penetration of the crater g, the shape of the crater, and the temperature of the bottom.

A rigorous formulation of the problem should allow simultaneously for the processes occurring both in the bulk of the metal and also in the flow of the produced metal vapor [1].

In the present work we set ourselves the aim of investigating whether the frontal method described in [2, 3] could be applied to the analysis of only a few factors, more specifically, to finding the stationary shape and the rate of motion of the evaporation front (the bottom of the crater). Processes connected with the condensation of the vapor and with the flow of heat to the side walls are disregarded. It has been shown [1] that these processes become determinative when the depth of the crater is much greater than its diameter. The method applied in the present paper assumes that the determinative process is the motion of the evaporation front in the metal; accordingly, it is applicable until such time as the depth of the crater becomes comparable with its diameter.

The problem is solved in the stationary approximation, i.e., it is assumed that an evaporation front of unchanging configuration F(r) moves at a constant rate g along the axis of the crater. The temperature of the metal near the front then depends only on the variable ξ [2, 3]:

$$\xi = V(x,\ r) + gt. \tag{1}$$

We suppose that the function V(x, r) can be written in the form V(x, r) = x − f(r), where the equation x − f(r) = const is the equation of an isotherm in a moving system of coordinates [f(r) is an unknown function]. Transforming the equation of thermal conductivity

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial x^2}\right)$$

to the variable ξ, we obtain

$$g = a\left(pf'^2 - f'' - \frac{1}{r}f' + p\right), \tag{2}$$

where

$$p = \frac{d^2T}{d\xi^2}\Big/\frac{dT}{d\xi} = \text{const}. \tag{3}$$

Integrating (3) gives

$$\frac{dT}{d\xi} = T_1 p e^{p\xi} \text{ and } T(\xi) = T_1 e^{p\xi},$$

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TABLE 1. Constants of Stationary Evaporation Process [$b = 2.5 \cdot 10^{-2}$ cm; $q_0/q(b) = 2$]

q_0, W/cm²	T_1, °K	T_2, °K	g, cm/sec	p, cm⁻¹	ξ_2, cm	h, cm	r_1, cm
Copper target							
10^6	4006	3898	8,25	59,0	$4{,}60 \cdot 10^{-4}$	$1{,}84 \cdot 10^{-2}$	$4{,}37 \cdot 10^{-2}$
10^7	5776	5413	187	204	$3{,}18 \cdot 10^{-4}$	$7{,}10 \cdot 10^{-3}$	$2{,}91 \cdot 10^{-2}$
$2 \cdot 10^7$	6414	5966	379	365	$1{,}98 \cdot 10^{-4}$	$4{,}25 \cdot 10^{-3}$	$2{,}72 \cdot 10^{-2}$
10^8	8466	7755	1773	1617	$5{,}43 \cdot 10^{-5}$	$1{,}13 \cdot 10^{-3}$	$2{,}55 \cdot 10^{-2}$
Aluminum target							
10^6	2989	2862	31,1	94,9	$4{,}58 \cdot 10^{-4}$	$1{,}23 \cdot 10^{-2}$	$3{,}46 \cdot 10^{-2}$
10^7	4084	3805	370	539	$1{,}31 \cdot 10^{-4}$	$2{,}74 \cdot 10^{-3}$	$2{,}65 \cdot 10^{-2}$
$4 \cdot 10^7$	5085	4686	1400	1986	$4{,}11 \cdot 10^{-5}$	$8{,}54 \cdot 10^{-4}$	$3{,}04 \cdot 10^{-2}$
10^8	6040	5515	3306	4683	$1{,}94 \cdot 10^{-5}$	$3{,}99 \cdot 10^{-4}$	$2{,}52 \cdot 10^{-2}$
Iron target							
10^6	4898	4655	17,2	156	$3{,}27 \cdot 10^{-4}$	$6{,}39 \cdot 10^{-3}$	$3{,}00 \cdot 10^{-2}$
10^7	6427	6007	175	1179	$5{,}74 \cdot 10^{-5}$	$1{,}08 \cdot 10^{-3}$	$2{,}56 \cdot 10^{-2}$
$5 \cdot 10^7$	8113	7507	824	5505	$1{,}41 \cdot 10^{-5}$	$2{,}73 \cdot 10^{-4}$	$3{,}01 \cdot 10^{-2}$
10^8	9130	8400	1589	10613	$7{,}86 \cdot 10^{-6}$	$1{,}53 \cdot 10^{-4}$	$2{,}51 \cdot 10^{-2}$

where $T_1 = T|_{\xi=0}$ is the temperature of the central point of the evaporation front.

From the condition $T_1|_{\xi \to -\infty} \to 0$, we obtain $p > 0$, and, accordingly, let us write $p = B^2$. In the case of a light flux of infinitely large radius it is clear that $f' \equiv f'' \equiv 0$ (planar problem), when $p = B^2 = g/a$. In the case of a light beam of finite radius the temperature along the axis $(x + gt)$ must decrease more rapidly, so that $B^2 > g/a$. Let us write

$$B^2 - \frac{g}{a} = A^2;\ z = ABr;\ f' = -\frac{1}{B^2} \cdot \frac{y'}{y}.$$

Equation (2) can then be written in the form

$$\frac{d^2y}{dz^2} + \frac{1}{z} \cdot \frac{dy}{dz} + y = 0. \quad (4)$$

For an isotherm we have the obvious condition $f'(r)|_{r=0} = 0$, so that we obtain

$$f' = \frac{A}{B} \cdot \frac{J_1(ABr)}{J_0(ABr)} \quad (5)$$

and

$$f(r) = -\frac{1}{B^2} \ln (J_0(ABr)) - \frac{1}{B^2} \ln \frac{T_1}{T}, \quad (6)$$

where T is the temperature of the given isotherm.

Expressions (5) and (6) are, strictly speaking, valid only in an infinitely thin layer adjacent to the evaporation front [2, 3]. At a finite distance from the front they can be regarded as holding with a certain accuracy, while for

$$r \gtrsim \frac{2{,}4}{AB},$$

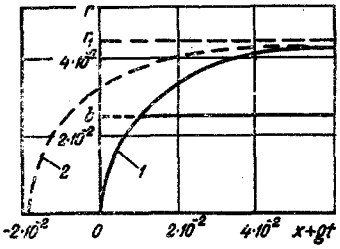
it is readily seen that these expressions lose significance. This situation does not, however, impose any limitations on finding the characteristics of the front, since, as will be seen from the subsequent discussions, all that we require to know is the temperature in an infinitely thin layer adjacent to the front.

The condition of energy balance on the evaporation front has the form

$$-\lambda \frac{\partial T}{\partial n}\bigg|_{x+gt=F(r)} = -q(r) \cdot \cos(\bar{n}, x) - \gamma v \Delta H, \quad (7)$$

where γ is the density of the metal; $v = -g \cos(\bar{n}, x)$ is the velocity of motion of a given point of the front along the normal to the front surface; the quantity

$$\Delta H = L - \frac{RT_1}{2\mu};$$



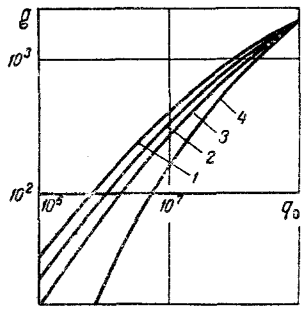


Fig. 1 Fig. 2

Fig. 1. Profile of bottom of crater under stationary evaporation conditions. Copper: $q_0 = 10^6$ W/cm^2; $b = 2.5\cdot10^{-2}$ cm; $q_0/q(b) = 2$. Curve 1 shows evaporation front; curve 2 shows the isotherm with melting temperature; r is in cm.

Fig. 2. Dependence of rate of motion of evaporation front g on energy density of laser beam q_0 for copper. Curve 1 corresponds to planar evaporation front; 2) $b = 1.25\cdot10^{-2}$ cm, $q_0/q(b) = 1$; 3) $b = 5\cdot10^{-2}$ cm, $q_0/q(b) = 2$; 4) $b = 1.25\cdot10^{-2}$ cm, $q_0/q(b) = 2$. g, cm/sec; q_0, W/cm^2.

L is the latent heat of evaporation of the metal; μ is the atomic weight of the metal; and

$$\cos(\bar{n}, x) = \frac{-1}{\sqrt{1+F'^2}}.$$

We transform Eq. (7) to the variable ξ, remembering that for $F = x + gt$ the quantity $\xi = F(r) + (1/B^2)\ln J_0(ABr)$:

$$\lambda T_1 B^2 J_0(ABr)\, e^{B^2F(r)}(1 + f'F') = q(r) - \gamma g\Delta H. \tag{8}$$

At the point $r = 0$ we have $F(0) = 0$; $q(0) = q_0$; $J_0(0) = 1$; $f'(0) = F'(0) = 0$, so that

$$T_1\lambda B^2 = q_0 - \gamma g\left(L - \frac{RT_1}{2\mu}\right), \tag{9}$$

i.e., we have an equation interrelating the unknown quantities T_1, g, and B. A second relationship between these constants comes from the kinetics of evaporation, the appropriate equation (following [1] and [4]) being taken in the form

$$v = v_0 e^{-\frac{L\mu}{RT}}, \tag{10}$$

where

$$v_0 = \bar{c}(3/4\pi)^{1/3}, \tag{10'}$$

where $\bar{c}$ is the mean velocity of sound in the metal, and T is the temperature of the given point of the front. At the point $r = 0$ we have

$$g = v_0 e^{-\frac{L\mu}{RT_1}}. \tag{11}$$

A third equation is obtained from (8) on setting $r = b$. We assume (as is done, for example, in [1]) that molten metal is completely removed from the side walls of the crater by the flow of vapor. The side wall then coincides with the isotherm corresponding to the temperature of melting T_m, and it is natural to suppose that the evaporation front at $r = b$ smoothly goes over to the isotherm, i.e., that

$$F'(b) = f'(b). \tag{12}$$

Utilizing (9), we can then bring Eq. (8) to the form

$$\lambda B^2\left[T_1 - T_2\left(1 + \frac{A^2}{B^2}\cdot\frac{J_1^2(ABb)}{J_0^2(ABb)}\right)\right] = q_0 - q(b), \tag{13}$$

where the temperature of the outermost points of the front is given by the expression

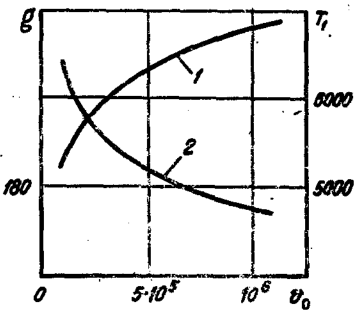


Fig. 3. Dependence of constants for stationary evaporation conditions on the parameter v_0 for copper: $q_0 = 10^7$ W/cm^2; $b = 2.5 \cdot 10^{-2}$ cm; $q_0/q(b) = 2$. Curve 1 shows the velocity of motion of the front g; curve 2 shows the temperature of the center of the bottom of the crater. g is in cm/sec; v_0 is in cm/sec; T is in °K.

$$T_2 = \frac{L\mu}{R \ln \left(v_0 \sqrt{1 + \frac{A^2}{B^2} \cdot \frac{J_1^2(ABb)}{J_0^2(ABb)}} \Big/ g \right)}. \tag{14}$$

Eliminating g and B from Eqs. (9), (11), and (13) gives a transcendental equation for the temperature of the central point of the front T_1 which can be solved with the aid of a computer.

Once T_1, B, and g have been found, Eq. (8) can be regarded as a differential equation defining the configuration of the evaporation front F(r). Utilizing (9), Eq. (8) can be written in the form

$$F'(r) = \frac{B\left[\left(1 - \frac{q_0 - q(r)}{\lambda B^2 T_1}\right) e^{-B^2 F(r)} - J_0(ABr)\right]}{AJ_1(ABr)}. \tag{15}$$

The function F(r) determines the shape of the bottom of the crater. The configuration of the side walls is determined by the equation

$$x = -\frac{1}{B^2} \ln\left(\frac{T_1 J_0(ABr)}{T_m}\right),\ b < r < \frac{2.4}{AB} \tag{16}$$

in the moving system of coordinates.

Like Eq. (6), expression (16) is approximate, being satisfied with decreasing accuracy the larger the value of x + gt. This is obvious from a physical point of view, since expression (16) makes no allowance for interaction of the metal with the flow of vapor.

Numerical calculations were made for copper, aluminum, and iron in the range of energy densities of the laser beam from 10^6 to 10^8 W/cm^2.

The results are presented in Figs. 1-3.

It can be seen from Table 1 that in all cases the distance along the X axis between the isotherms T_1 and T_2 is small:

$$\xi_2 = \frac{1}{B^2} \ln \frac{T_1}{T_2} \ll b.$$

There is no need to solve Eq. (15), since for any form of monotonically decreasing function q(r) the evaporation front is located between the specified isotherms. The function F(r) effectively coincides with the function (1/p) ln $(J_0(ABr))$; to find the constants we do not require the function q(r), but only its values at two points: $q(0) = q_0$ and q(b) subject to the condition that q(r) monotonically decreases.

A comparison of the cited values of the rate g with data from the monograph of Anisimov et al. [1] shows that the obtained results are in satisfactory agreement with experiment.

By way of example we have depicted in Fig. 1 the profile of the bottom of the crater in a copper target for $q_0 = 10^6$ W/cm^2 and $q_0/q(b) = 2$. It can be seen from Fig. 2 that the velocity of motion of the front g depends significantly on its curvature. Even when the energy density is uniformly distributed over the cross section of the beam, i.e., when $q_0 = q(b)$ (curve 2), the velocity g is appreciably less than the velocity of a planar front (curve 1). While if $q_0/q(b) = 2$ (curves 3 and 4), neglecting the curvature of the front leads to almost a 10-fold error in the determination of g in the range 10^6–10^7 W/cm^2.

Figure 3 shows the dependence of the rate of motion of the front g and the temperature T_1 on the parameter v_0. As pointed out in [1], formula (10') very roughly determines the pre-exponential factor in the kinetic equation. The calculations show, however, that the magnitude of this parameter has only a small effect on the results of the computations; varying v_0 by an order (from 10^5 to 10^6 cm/sec) changes the velocity g only by around 5%.

NOTATION

T_1, temperature of the center of the bottom of the crater; T_2, temperature of the outermost points of the front (at r = b); g, stationary rate of deepening of the crater; $p = (d^2T/d\xi^2)/(dT/d\xi)$; $\xi_2 = (1/B^2) \times \ln(T_1/T_2)$, distance along the X axis from the central point of the front to the isotherm with temperature T_2; $h = (1/B^2) \ln (T_1/T_m)$, depth of the molten layer along the X axis; $r_1 = (2{,}4/AB)$.

LITERATURE CITED

1. S. I. Anisimov, Ya. A. Imas, G. S. Romanov, and Yu. V. Khodyko, Influence of High-Power Radiation on a Metal [in Russian], Izd. Nauka (1970).
2. A. S. Predvoditelev, "On the general theory of wave processes," in: Physical Gasdynamics and the Properties of Gases at High Temperatures [in Russian], Izd. Nauka (1964).
3. V. M. Eroshenko, A. V. Morar, and Yu. N. Terent'ev, "Frontal character of transition surface of a substance in interaction with an external medium," in: Properties of Gases at High Temperatures [in Russian], Izd. Nauka (1967).
4. J. P. Hirth and G. M. Pound, Progress in Metal Physics: Condensation and Evaporation, Pergamon (1963).

GROUP PROPERTIES OF THE NONLINEAR HEAT-CONDUCTION EQUATION AND THE SOLUTION OF INVERSE PROBLEMS

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We develop a numerical—experimental method of determining the thermophysical properties of materials in which we use group-invariant solutions of the nonlinear heat-conduction equation. We study the stability of a class of such solutions.

In [1] Ovsyannikov examined the problem of the group classification of the equation

$$\frac{\partial u}{\partial \tau} = \frac{\partial}{\partial x}\left(f(u)\,\frac{\partial u}{\partial x}\right), \tag{1}$$

i.e., the problem of determining the fundamental group admitted by Eq. (1) for various forms of the function f of the unknown solution.

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